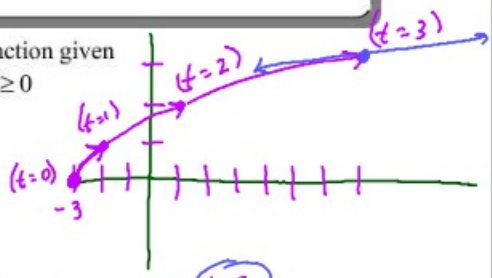
What you'll Learn About
How to take the derivative of functions in Parametric Form

t	x	y
0	-3	0
1	-2	1
2	1	2
3	6	3

Graph the parametric function given

A) $x = t^2 - 3$ $y = t$ $t \geq 0$



B) Find the derivative of the function at ~~t=3~~ t=3.

$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{1}{2t}$

$x = t^2 - 3$ $y = t$

$\frac{dx}{dt} = 2t$ $\frac{dy}{dt} = 1$

$\boxed{\left.\frac{dy}{dx}\right|_{t=3} = \frac{1}{6}}$

C) Find the equation of the tangent line at t=1

$y = y_1 + m(x - x_1)$

$x = 3t$ $y = 9t^2$

Point (x, y)

$\frac{dy}{dx} = \frac{18t}{3} = 6t$

$(3, 9)$

$\left.\frac{dy}{dx}\right|_{t=1} = 6$

$\boxed{y = 9 + 6(x - 3)}$

D) Find the equation of the tangent line at $\theta = \frac{\pi}{4}$

$x = \cos\theta$ $y = \sin\theta$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\boxed{y = \frac{\sqrt{2}}{2} - 1\left(x - \frac{\sqrt{2}}{2}\right)}$

$\frac{dy}{dx} = \frac{\cos\theta}{-\sin\theta} =$ $\left.\frac{dy}{dx}\right|_{\theta = \pi/4} = \frac{\cos \pi/4}{-\sin \pi/4} = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$

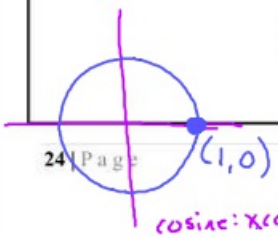
E) Find the equation of the tangent line at $t = \pi$

$x = \sec^2(2t) - 1$ $y = \tan(2t)$

$x = \sec^2(2\pi) - 1$ $y = \tan 2\pi$ $(0, 0)$

$x = 1^2 - 1$ $y = \frac{0}{1} = \boxed{0}$

$x = 0$



cosine: x-coord
sine: y-coord

$\sin = \frac{1}{\csc}$

$\cos = \frac{1}{\sec}$

$\tan = \frac{1}{\cot}$

$x = [\sec(2t)]^2 - 1$

$\frac{dx}{dt} = 2\sec(2t) \cdot \sec(2t)\tan(2t) \cdot 2$

$\frac{dy}{dt} = \sec^2(2t) \cdot 2$

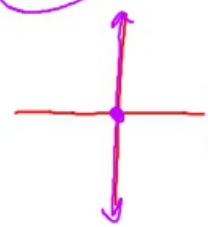
$\frac{dy}{dx} = \frac{2\sec^2(2t)}{4\sec^2(2t)\tan(2t)}$

$\frac{dy}{dx} = \frac{1}{2\tan(2t)}$

Point $(0, 0)$

$\left.\frac{dy}{dx}\right|_{t=\pi} = \frac{1}{2\tan(2\pi)} = \frac{1}{2(0)} =$ UND (SLOPE)

vertical tangent



$\boxed{x = 0}$

$\boxed{y = 64 + 12(x-1)}$

A curve C is defined by the parametric equations $x = t^2 - 4t + 1$ and $y = t^3$. Determine the equation of the line tangent to the graph of C at the point (1, 64)?

$\frac{dy}{dx} = \frac{3t^2}{2t-4}$

$\left.\frac{dy}{dx}\right|_{t=4} = \frac{48}{4} = 12$

$x = t^2 - 4t + 1$ $\quad y = t^3$

$1 = t^2 - 4t + 1$ $\quad 64 = t^3$

$0 = t^2 - 4t$ $\quad 4 = t$

$0 = t(t-4)$

$t = 0$ $\quad t = 4$

Determine the horizontal and vertical tangents for the parametric curve

A) $\boxed{x = 1 - t}$ $\boxed{y = t^2 - 4t}$

$\frac{dy}{dx} = 0$ ← Horizontal Tangent

$\frac{dy}{dt} = 0$ $\quad y = t^2 - 4t$

$2t - 4 = 0$

$t = 2$

Vertical Tangent $\frac{dy}{dx} \to$ UND

$\frac{dx}{dt} = 0$ $\quad x = 1 - t$

$\frac{dx}{dt} = -1$ $\quad$ No V.T.

B) $x = \cos\theta$ $y = 2\sin(2\theta)$ $\quad [0, 2\pi)$

H.T $\frac{dy}{dt} = 0$

$\frac{dy}{dt} = 2\cos(2\theta) \cdot 2$

$0 = 4\cos 2\theta$

When is x-coord zero? ← $0 = \cos(2\theta)$

$2\theta = \frac{\pi}{2}$ $\quad 2\theta = \frac{3\pi}{2}$

$\theta = \frac{\pi}{4}$ $\quad \theta = \frac{3\pi}{4}$

VT $\frac{dx}{dt} = 0$

$\frac{dx}{dt} = -\sin\theta$

$0 = -\sin\theta$

$0 = \sin\theta \to$ y-coord $= 0$

$\theta = 0, \pi$